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**THE TEMPORAL CONSISTENCY OF  
THE EFFECTS OF STRUCTURE AND  
CONTENT ON SPATIAL DEDUCTIVE  
REASONING.**

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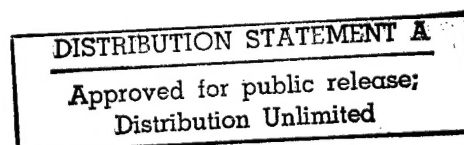
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## Executive Summary

In air, ground, or naval navigation, military operators are required to represent the relative location of entities in three dimensions. They may construct their mental representation from sentences describing the relative location of entities, or from diagrams depicting these locations. The entities of the sentences and diagrams may be symbolised using either nouns or images. The sentences and diagrams may further specify the relative locations of entities according to euclidian (e.g., number of dimension), projective (e.g., orientation, direction), and topological (e.g., proximity, enclosure) relations. From these mental representations, military operators may be required to deduce the relative location of entities not explicitly related, from the relationships that have been described or depicted among the set of entities.

How do humans represent and reason about sentences and diagrams which specify geometrical contents? Cognitive scientists have proposed two opponent theoretical views to account for the mental representation and processes that underlie formal reasoning. These theoretical views suggest: (1) that there are two fundamentally distinct processes that underlie spatial reasoning; and (2) that these would be the same irrespective of an argument's symbolic representation. However, in the case of spatial deductive reasoning, there is no empirical evidence that supports the theories' claims. One reason for the lack of empirical evidence arises from the long standing tradition of using sentences to investigate formal reasoning. However, humans normally make deductions from analogue representations which depict, to a certain degree, the structure of the relations among entities. Such is the case in mechanical, electronic, or medical diagnostic activities. It is also the case for spatial deductive reasoning. Yet, little is known about the nature of the processes that underlie formal reasoning from different modes of symbolic representation and whether their effects are temporally consistent.

The overall objective of this study is to elucidate the temporal consistency of the effects of the structure (symbolic and logical) and content (symbolic and geometrical) of an argument on the mental representation and processes that underlie spatial deductive reasoning. We will investigate the effects of these factors mainly by comparing the opposing predictions made by two opponent theories of formal reasoning: the Formal Rules theory and the Mental Models theory.

Thirty-seven subjects solved 48 spatial reasoning problems which varied in their *logical structure*, that is, by the order of the entities in the premise sets, and their *geometrical content*, that is, by the number of dimensions (2D, 3D), orientations (horizontal, vertical), and directions (right/left, bottom/top) specified in the premise sets. For half of the problems, subjects were to deduce the relative location of an object pair (EB) in two dimensions. For the other half, subjects were to deduce the relative location of an other object pair (ED) in either one dimension, for the 2D condition, or three dimensions for the 3D condition. Questions pertaining to each dimension were asked consecutively, thus requiring deductions over time but within the bounds of working memory.

We varied the symbolic representation of the problems according to their *symbolic structure* using sentences or diagrams to represent the relations among entities, and their *symbolic content* using nouns or

images to represent the entities. We addressed temporal consistency by measuring the effects of the above variables on the responses and response times obtained for the successive questions that were asked for the 2D and the 3D conditions.

Spatial deductions were reliably easier to make from arguments displayed as diagrams than as sentences indicating that subjects were converting sentences into mental models. This interpretation was further supported by the fact that problems based on multiple mental models were more difficult than those based on one mental model. However, the effect of the structure (symbolic and logical) of the arguments did not remain significant throughout the successive questions. The symbolic content of the arguments had no reliable effect on spatial deductions ( $p > 0.01$ ). Thus, as predicted by the two theories of spatial reasoning, the effect of the symbolic structure of the arguments prevailed over the lexical tokens used to symbolise the arguments' entities.

Spatial deductions varied reliably with the number of dimensions and directions specified in the premise sets despite the fact that the geometrical relations involved identical formal derivations. The effects of the geometrical relations were also temporally significant and consistent. The results suggest that subjects constructed mental models which reproduced geometrical relations relative to spatial reference frames. Together, the effects of the structure (symbolic and logical) and content (symbolic and geometrical) of an argument thus supported the Mental Models theory's predictions while refuting those of the Formal Rules theory.

### Abstract

The general aim of this study is to elucidate the temporal consistency of the effects of the structure and content of an argument on spatial deductive reasoning. We will investigate the effects of these factors mainly by comparing the opposing predictions made by the Formal Rules theory and the Mental Models theory of spatial deductive reasoning.

Thirty-seven subjects solved 48 spatial reasoning problems which varied by their *logical structure*, that is, by the order of the entities in the premise sets, and their *geometrical content*, that is, by the number of dimensions (2D, 3D), orientations (horizontal, vertical), and directions (right/left, bottom/top) specified in the premise sets. For half of the problems, subjects were to deduce the relative location of an object pair (EB) in two dimensions. For the other half, subjects were to deduce the relative location of an other object pair (ED) in either one dimension, for the 2D condition, or in three dimensions for the 3D condition. Questions pertaining to each dimension were asked consecutively thus requiring deductions over time but within the bounds of working memory.

We varied the symbolic representation of the problems according to their *symbolic structure* using either sentences or diagrams to represent the relations among entities, and their *symbolic content* using either nouns or images to represent the entities. We addressed temporal consistency by measuring the effects of the above variables on the responses and the response times obtained for the consecutive questions.

Spatial deductions were reliably easier to make from diagrams than from sentences, thus confirming the Mental Models theory. However, diagrams facilitated spatial reasoning only during the initial stage of the process of deduction. Once subjects had consolidated their mental models of the spatial relations, their deductions were just as easy to make from sentences than from diagrams ( $p > 0.01$ ).

The symbolic content of the arguments had no reliable effect on spatial deductions ( $p > 0.01$ ). Thus, as predicted by the two theories of spatial reasoning, the effect of the symbolic structure of the arguments prevailed over the lexical tokens used to symbolise the arguments' entities. The effect of logical structure provided additional evidence that subjects constructed mental models, and thus converted sentences into mental models. However, the effect of logical structure did not persist throughout the successive questions. Spatial deductions were reliably easier to make from the 2D condition than from the 3D one, and the differences remained significant throughout the successive questions ( $p < 0.01$ ). The effects of direction on spatial deductions also remained significant and consistent throughout the successive questions ( $p < 0.01$ ). The effects of the geometrical relations thus supported the Mental Models theory while elucidating the structural properties of mental models. The results suggest that mental models reproduce the euclidan and projective relations among the entities according to spatial reference frames. Altogether, the effects of the structure (symbolic and logical) and content (symbolic and geometrical) of an argument supported the Mental Models theory's predictions while refuting those of the Formal Rules theory.

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## 1. Introduction

In a previous study (Boudreau & Pigeau, 1997), we investigated the effects of the structure (symbolic and logical) and content (symbolic and geometrical) of an argument on the mental representation and processes that underlie spatial deductive reasoning. We elucidated the effects of these factors by comparing the opposing predictions made by the following two theories of spatial deductive reasoning: Hagert's Formal Rules theory (Hagert, 1985; Hagert & Hansson, 1983, 1984) which is one of the Formal Rules theories, and Johnson-Laird's Mental Models theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991, 1993). We also investigated the effects of geometrical content in view of the spatial reference frame theory (Logan, 1995).

The study of Boudreau and Pigeau (1997) had two general objectives. The first was to elucidate the effects of the symbolic structure and the symbolic content of an argument on the mental representation and processes that underlie spatial deductive reasoning. The first factor, *symbolic structure*, referred to the structure of the relations among entities. There were two types of symbolic structure, sentences which described relations, and diagrams which depicted the relations. The second factor, *symbolic content*, referred to the lexical tokens, words and images, that symbolised the entities of the arguments.

The second general objective of Boudreau and Pigeau (1997) was to elucidate the effects of the *logical structure* and the *geometrical content* of an argument on the mental representation and processes that underlie spatial deductive reasoning. We investigated the effects of logical structure by varying the referential determinacy of the entities' order in the arguments. Referential determinacy specified the extent to which the order of referents (or entities) was determinate or unambiguous. In the following example -- B directly left of A; C directly left of B -- the order of the referents is determinate in that each entity occupies a unique position relative to another, thus yielding one layout namely, CBA. The geometrical content, which was of interest, specified the relative location of entities according to euclidean (number of dimensions: 2D and 3D) and projective (orientation, direction) relations.

We investigated the effects of the structure (symbolic and logical) and content (symbolic and geometrical) of an argument on three dependant variables: the premises' display times, the responses, and the response times. Each of these variables was subject paced. We measured the premise display times to elucidate the nature of the mental representation of spatial relations and we measured the responses and the response times to determine the process of deduction itself.

Each problem included up to three questions depending on the dimensional condition. We analysed the responses and the response times obtained for the first question of each problem to allow comparisons of the results of this study with those of related studies (Barwise & Etchemendy, 1992; Bauer & Johnson-Laird, 1993; Byrne & Johnson-Laird, 1989; Tabachneck & Simon, 1992) where problems included only one question. The results of the study confirmed the Mental Models theory's predictions while refuting those of the Formal Rules theory. The problem remained, however, to specify the extent to which the observed effects persisted over time despite working memory limitations. The issue is particularly important in the



case of diagrams which, in contrast to sentences, have been considered to facilitate deductive reasoning. However, if this effect has been observed for the duration of one question (Barwise & Etchemendy, 1992; Bauer & Johnson-Laird, 1993; Boudreau & Pigeau, 1997; Tabachneck & Simon, 1992) will the effect persist over time even within working memory limitations? To address this issue and related ones, we will in this study elucidate the temporal consistency of the effects of the structure and content of an argument on the processes of spatial reasoning. More specifically, we will determine whether the processes observed for one question persisted across the successive questions that were asked for the 2D and the 3D conditions. In the following section, we will present an overview of the effects of the structure and content of an argument (Boudreau & Pigeau, 1997) with respect to predictions made by the Formal Rules theory versus those made by the Mental Models theory of spatial deductive reasoning.

### 1.1. Effects of the Symbolic Representation of an Argument on the Mental Representation and Processes of Spatial Reasoning

The Formal Rules theory and the Mental Models theory of spatial reasoning claim that the structure of the mental representation and processes of spatial reasoning should be the same irrespective of the symbolic representation of an argument. However, both theories suggest that there are two fundamentally distinct mental representations and processes that underlie deductive reasoning. The Formal Rules theory posits that all arguments, whether symbolised as sentences or as diagrams, are represented as mental propositions which reveal an argument's logical structure. For instance, the propositional representation of an argument symbolised as sentences such as:

The star is to the left of the cross.

The circle is to the left of the star.

or as corresponding diagrams:

\* +.

O \*.

will have the same logical structure expressed in first-order predicate logic

1. Left (star(y), cross(x)).

2. Left (circle(z), star(y)).

Formal rules of inference such as:

1'. Left (y, x) and 2'. Left (z, y) implies 3'. Left (left (z, y) x).

are then applied to the corresponding mental propositions:

1. Left (star(y), cross(x)). 2. Left (circle(z), star(y)).

in order to derive a conclusion:

3. Left (left(circle, star) cross)<sup>1</sup> irrespective of the argument's geometrical content.

---

<sup>1</sup> The conclusion means that the circle is to the left of the star and the cross.

The alternative Mental Models theory repudiates the Formal Rules account arguing that all arguments are represented as mental models based on the premises' logical structure and geometrical content. Thus, a mental model of sentences such as:

The star is to the right of the cross.

The circle is to the left of the star.

or of the corresponding diagrams:

Cross    Star.

Circle   Star.

will have the same geometrical structure:

O   +   \*

irrespective of an argument's symbolic representation. Conclusions are drawn and validated by inspecting these mental models without the use of formal rules of inference or even content-specific rules (Cheng & Holyoak, 1985).

The two theories thus posit fundamentally different mental representation and processes of spatial reasoning. The theories also make opposing predictions regarding the effects of the symbolic representation of an argument on the levels of information processing required, in working memory, to construct the mental representation and processes of spatial reasoning.

#### 1.1.1. Effects of the Symbolic Structure

The Formal Rules theory predicts that mental propositions should be easier to construct from sentences than from diagrams as mental propositions are based on the syntactical structure of sentences. Formal rules of inference should also be easier to apply when questions are structured as sentences than as diagrams since subjects would have to convert the latter into sentences prior to deriving a conclusion. The Mental Models theory makes the opposite predictions arguing that mental models are based on the structure of diagrams depicting spatial relations. Consequently, if an argument is displayed as sentences depicting relations, subjects would first convert each sentence into a mental layout which they would then integrate into a mental model.

The effects of the symbolic structure of the arguments confirmed the Mental Models theory's predictions while refuting those of the Formal Rules theory. The premise display times showed that mental representations were reliably easier ( $p < 0.01$ ) to construct from premises displayed as diagrams than as sentences thus suggesting that subjects constructed mental models of the spatial relations.

The construction of mental models also required different levels of processing in working memory depending on the arguments' symbolic representation. When arguments were displayed as sentences, subjects would form a mental proposition that is close to the surface form of the sentence (Johnson-Laird, 1983; Mani & Johnson-Laird, 1982). They would then use this mental representation as a basis to construct a mental layout analogous to the structure of a diagram. The progressive integration of these mental layouts

would then yield a mental model. Hence, the representation of spatial descriptions as mental models necessarily required more processing time than their encoding as mental propositions. In contrast, when arguments were displayed as diagrams, subjects would directly represent each diagram as a mental layout and integrate these mental layouts into a mental model. Bypassing the conversion of sentences into diagrams appears to reduce the load on working memory, and to accelerate the construction of mental models. This interpretation does not exclude the possibility that subjects interpreted the diagrams as sentences, while maintaining the diagrams in working memory. But if so, this process would have occurred more rapidly than the interpretation of sentences as diagrams. The results support the notion that diagrams are not merely encoded as mental propositions as argued by the Formal Rules theorists (Braine, 1978; Braine & O'Brien, 1991; Hagert, 1985; Piaget, 1972; Rips, 1983).

#### 1.1.2. Effect of the Symbolic Content

The Formal Rules theory describes mental propositions as having referentially arbitrary properties since humans would use nouns or arbitrary signs as lexical tokens to symbolise the entities of the propositions. In contrast, the Mental Models theory describes mental models as having referentially isomorphic properties as nouns and images would be used as lexical tokens. If humans construct mental propositions, these should be easier to symbolise using nouns as lexical tokens rather than images whereas if humans construct mental models, their entities may just be as easy to symbolise using nouns or images.

The use of nouns or images as lexical tokens within diagrams affected neither the mental representation nor the process of spatial reasoning ( $p > 0.01$ ). However, the use of images within sentences increased the difficulty of representing and reasoning from such symbolic contents because of the increased mental workload required to solve the incongruence between the representation of images within sentences (Nelson, Reed, & McEvoy, 1977; Paivio, 1971, 1986; Pavio & Csapo, 1969). Nouns were reliably easier to represent when imbedded within diagrams than within sentences. This result suggests that subjects were directly ordering nouns as mental layouts thus confirming the Mental Models theory.

#### 1.2. Effects of the Logical Structure of an Argument on the Mental Representation and Processes of Spatial Reasoning

While the effects of the symbolic structure provided an alternative method of pitting the opposing predictions made by the Formal Rules theory and the Mental Models theory, the effects of logical structure allowed us to further specify whether in fact subjects constructed mental models.

We manipulated the arguments' logical structure using three levels of referential determinacy: problems of type 1, type 2, and type 3. If subjects construct mental models, then problem type 2, based on two mental models, should be more difficult to represent and reason from than problems of type 1 and type 3 based on one mental model. In contrast, if subjects construct mental propositions, the premises of all three problem types should be equally easy to represent as they involve the same number of mental propositions, and thus same amount of mental workload. Moreover, deductions from problems of type 3 should be more

difficult than from those of type 1 and type 2 as the former involve a longer formal derivation than the latter two based on identical ones.

The overall effect of logical structure supported the Mental Models theory's predictions as problems which involved two mental models took more time ( $p < 0.01$ ) to represent and solve than those (problems of type 1 and type 3) which involved only one mental model. The results were highly reliable although all three problem types involved the same number of mental propositions, and although problems of type 2 required a shorter formal derivation than those of type 3. These results generalised across the symbolic structure and the symbolic content of the arguments ( $p > 0.01$ ). They were also consistent across the arguments' geometrical content ( $p > 0.01$ ).

### 1.3. Effects of the Geometrical Content of an Argument on the Mental Representation and Processes of Spatial Reasoning

We assessed the effects of geometrical content by varying the euclidian (number of dimensions) and projective (orientation and direction) relations among the entities. The Formal Rules theory predicts that the geometrical content of the arguments should not affect their difficulty as the length of the formal derivations were the same across contents. However, the Mental Models theory predicts that the structure of the mental models should be identical to the structure of the relations among the entities as they are perceived or conceived. The difficulty of spatial reasoning should thus vary with the euclidian and projective relations among the entities. The differential effects of the euclidian relations would indicate that subjects are constructing mental models relative to a set of three coordinate axes which specify the basis of spatial reference frames. The effects of the projective relations would help elucidate two basic parameters of the spatial reference frames, namely their orientation and direction (Logan, 1995).

The arguments' geometrical content systematically affected ( $p < 0.01$ ) the difficulty of spatial reasoning although the formal derivations were of the same length across contents. The results thus confirmed the Mental Models theory's predictions while elucidating the geometrical structure of the mental models. Thus, the difficulty of spatial reasoning increased systematically with the number of dimensions that subjects had to deduce from a mental model. This result indicates that mental models reproduce euclidian relations among entities in a way similar to which mental images reproduce metric relations (Jeannerod, 1994; Kosslyn, Ball, & Reiser, 1978). This functional similarity would be based on the partial isomorphisms that exists between perceptual structures and spatial-temporal structures (Johnson-Laird, 1983; Piaget & Morf, 1958).

The projective relations (orientation and direction) also reliably affected the difficulty of spatial reasoning ( $p < 0.01$ ). The effect of orientation on the premise display times indicated that mental models were systematically more difficult to construct from the vertical layouts than from the horizontal ones. These results had two implications. For the 2D condition, the effect of orientation suggests that the vertical axis was easier to represent relative to the horizontal axis than the opposite was true. For the 3D condition, the results indicate that the line of sight axis was easier to represent relative to the horizontal axis than to the vertical axis. However, orientation did not have a significant main effect on the difficulty of spatial

deductions. As predicted by the Content-Specific Rules theory (DeSoto, London, & Handel, 1965; Handel, DeSoto, & London, 1968), subjects also showed systematic patterns in constructing mental models from one set of directions rather than from the opposite one. Deductions were also reliably easier to make from the same set of directions from which subjects constructed mental models, that is, from right to left of the horizontal axis and from bottom to top of the vertical axis. The effects of the projective relations thus indicated that subjects' spatial reference frames had an orientation and direction (Logan, 1995) that directed conceptual attention during the construction of mental models. Logan's (1995) spatial reference frame hypothesis thus generalised to the process of spatial deductive reasoning while providing a complementary account of the effects of geometrical content which supported the Mental Models theory. In this study, we will determine whether the effects of structure (symbolic and logical) and content (symbolic and geometrical) were temporally consistent.

## 2. Method

### 2.1. Materials

#### 2.1.1. Spatial Deductive Problems

Each spatial deductive problem consisted of four premises and up to three questions. Each premise described or depicted the relative location of two entities. Together, the four premises formed a *set* which described or depicted the layout of five entities (circle, square, cross, diamond, and star). The subject's task was to determine, from a premise set, whether the relative location of two entities stated (or depicted) in a question was true or false.

We used two general factors to construct the problems' premise sets: *logical structure*, which pertained to the order of the objects in the premise sets, and *geometrical content* which pertained to the relationships among the objects. We varied the logical structure of the premises according to three levels of referential determinacy (problem type 1, problem type 2, and problem type 3). We specified the geometrical content of the premises using three variables: number of dimension (2 levels: 2D and 3D), orientation (2 levels: horizontal and vertical), and direction (2 levels: right/left for the horizontal orientation, and below/above for the vertical orientation). We generated a total of 24 premise sets from the above four variables ( $3 \times 2 \times 2 \times 2$ ). The 24 premise sets were presented twice, once for each of two question types (EB relation and ED relation). For each question type, subjects were asked to deduce the relative location of two entities (EB or ED) not explicitly related in any of the premises of a set. The number of questions requested for each question type varied between 1 and 3 depending on the dimensional condition. The experiment comprised a total of 48 problems representing a  $3 \times 2 \times 2 \times 2 \times 2$  within-subject factorial design. The number of questions requested for the question types constituted a nested variable.

Appendix A presents the variables and the resulting experimental problems. The appendix also describes the number of mental models, and the length of the formal derivation underlying the solution of

each problem. The length of the formal derivation is defined by the minimum number of inferential steps used in logic and are illustrated in the appendix.

We varied the *symbolic representation* of the problems according to their *symbolic structure*, using sentences or diagrams to represent the relations among entities, and their *symbolic content* using nouns or images to represent the entities. These variables were specified as two between-subject conditions.

### 2.1.1.1. Logical Structure of the Arguments

We addressed the effects of logical structure by varying the determinacy of the entities order in the arguments. Referential determinacy included three levels: problem type 1, problem type 2, and problem type 3.

Problems of type 1 specified the position of the objects unambiguously using the following order between the pairs of objects of each premise: A-B, C-B, D-B, C-E<sup>2</sup>. For example, the following problem -- 1) A right of B, 2) C left of B, 3) D below B, 4) C above E. Is E left of D? -- yields one mental model:

C B A

E D.

Problems of type 2 were the same as those of type 1, except that the order of the objects A-B was inverted to B-A. This inversion caused the location of A to become ambiguous relative to C, given the location of C and B. For example, the following problem -- 1) B right of A; 2) C left of B; 3) D below B; 4) C above E. Is E left of D? -- underlies two mental models:

a) C A B

b) A C B

E D

E D

According to the Formal Rules theory (Hagert, 1985; Hagert & Hansson, 1983, 1984), subjects do not require the first premise of the above two problem types to deduce the location of E relative to D or to B. Since the three remaining premises are identical then both problem types involve the same formal derivation (see Appendix A). The Formal Rules theory thus predicts that problems of type 1 and type 2 should be equally difficult.

Problems of type 3 were identical to those of type 1, except that the object D was related to A instead of B. This relation insured that the first premise would be required in the formal derivation of the ED relation. For example, the following problem -- 1) A right of B, 2) C left of B, 3) D below A, 4) C above E. Is E left of D? -- yields one mental model: C B A

E D.

Yet, the problem type involves a longer formal derivation than problem type 2, and should thus be more difficult. The Mental Models theory makes the opposite prediction.

---

<sup>2</sup> During actual experimental runs, all problems used the same set of five objects (circle, square, cross, diamond, and star) to control for any potential effects due to content other than those investigated in this study. However, we assigned the objects randomly to the entities A, B, C, D, and E, thus forming for each

#### 2.1.1.2. Geometrical Content of the Arguments

**Number of dimensions.** We varied the number of dimensions described (or depicted) in the premise sets according to two dimensional conditions: 2D and 3D. For the 2D condition, the premise sets described 2D layouts of entities along the horizontal and vertical axes; and for the 3D condition, the premise sets described 3D layouts along the horizontal, vertical, and line of sight axes.

The number of dimensions requested in a question type (ED relation and EB relation) depended on the dimensional condition. For the 2D condition, subjects were required to deduce the relative location of ED in one dimension (e.g., Is E left of D?) either along the horizontal or the vertical axis. They were also to deduce the relative location of EB in two dimensions along the horizontal and vertical axes. For the 3D condition, subjects were to deduce the ED relation in three dimensions, and that of EB in two dimensions (as for the 2D problems). Each question type stated the relative location of the objects one axis at a time, either for the horizontal, vertical, or the line of sight axis. Thus, subjects had to determine whether the relative location of the two objects was correct or incorrect for one axis at a time. For example, a question for the horizontal axis would be:

?

Square left of Diamond

The two question marks cued the subject that the sentence or diagram was a question without biasing a left to right reading for the diagrams.

**Orientation.** The premise sets described horizontal layouts and vertical layouts. The relative location of the entities A, B, and C, which formed the main axis of a layout, determined their orientation. The vertical layouts (V1 and V2) corresponded to a 90 degree clockwise rotation of the horizontal layouts (H1 and H2 respectively). The layouts were topologically equivalent.

Direction. The premise sets described (or depicted) the layouts starting with the entity A (or B) as their point of departure. Given that subjects assumed a fronto-parallel view relative to the entities, the premise sets specified the horizontal layouts from right to left, and left to right; and the vertical layouts from bottom to top, and top to bottom. The relative location of the entities A, B, and C, which formed the main axis of a layout specified the direction from which they were described (or depicted). For example, the following premise set -- 1) A right of B, 2) C left of B, 3) D below B, 4) C above E -- describes the position of the entities A, B, and C from right to left along an horizontal axis.

#### 2.1.1.3. Controls

To insure that each experimental problem generated one valid conclusion, we instructed the subjects to make three assumptions: (1) that the entities of a problem were directly in front of them, that is, in a fronto-parallel plane, (2) that the entities located on a given axis were collinear, that is "on the same straight line", and (3) that no two entity could occupy the same position at the same time.

subject different sets of lexical tokens.

It is important to recognise that without the above conditions, the problems yield an infinite number of formal derivations and mental models. Therefore, in order to carefully test the predictions of the Formal Rules theory and the Mental Models theory, these controls are essential. Moreover, subjects can formulate the above assumptions strictly in terms of propositions without using spatial reference frames.

### 2.1.2. Symbolic Representation of the Arguments

We varied the symbolic representation of the arguments according to their *symbolic structure* and their *symbolic content*. The symbolic structure of the arguments pertained to the mode, textual or graphical, in which we displayed the relations between the entities. Relations were displayed according to a *textual structure* and a *graphical structure*. The textual structure consisted of sentences which described the relative location of each pair of entities using a projective preposition (e.g., "left of", "in front of"). There were four sentences for each premise set. The graphical structure on the other hand consisted of diagrams which depicted each pair of entities according to their relative location without using projective prepositions (see section 1.1 for an example). There were four diagrams for each premise set. The symbolic content of the problems pertained to the lexical tokens, nouns or images, that symbolised the entities of each sentence or diagram.

For each symbolic structure, the content of each premise and question was displayed sequentially. The textual structure displayed each premise (and question) in three stages. First it presented only entity one (e.g., circle), then entity one and the projective preposition (e.g., circle left of), and finally the whole premise (e.g., circle left of triangle) or question. Entity one appeared in the center-left of the computer screen, the projective preposition in the center, and entity two in the center-right of the screen. The graphical structure displayed each premise (and question) in two stages. First it displayed only entity one (e.g., circle), then entity one and entity two (e.g., circle triangle), that is, the whole premise or question. Entity one appeared in the center of the computer screen, and entity two was placed relative to the first one according to the relationship, that is, left or right, above or below, and in front or behind. To illustrate the front/behind relationship, the entity in front appeared larger and brighter, and the entity behind appeared smaller and darker.

### 2.2. Procedure

Subjects participated individually in a sound-attenuated room equipped with a 386 PC for problem generation and a two-button mouse for responses. They received eight practice problems followed by the experimental problems. Subjects controlled the display time of each premise (and question), once it was completely presented, by pressing any key on the keyboard or one of two mouse buttons. They were instructed to proceed as quickly and as accurately as possible.

The construction of each premise (and each question), prior to being completely displayed on the computer screen, was done according to predefined temporal parameters:



- Total intrapremise time: 900 ms
- Total intraquestion time: 900 ms

Each sentence of the textual structure presented a delay of 450 ms between entity one and the relationship, and between the relationship and entity two, for example: star (450 ms) left of (450 ms) cross. Each diagram of the graphical structure presented two times the delay of 450 ms between the display of entity one and that of entity two, for example: star (900 ms) cross. The total intrapremise time was thus 900 ms for each symbolic structure.

After receiving a premise set, subjects were shown up to three consecutive questions that required them to validate a stated location between two objects (e.g., 'star left of the cross?'). The truth or falsity of a stated location was varied randomly for each subject, problem, and question. Each question remained displayed until the subject responded by selecting the left mouse button (yes: the relative location is true), the right mouse button (no: the relative location is false), or any key on the keyboard if they did not know the answer. These were treated as incorrect.

Subjects response times were measured to the nearest millisecond from the moment a question was completely displayed on the computer screen. A delay of 400 ms intervened between the answer to one question and the presentation of the next question (when more than one was asked). A delay of 8 seconds separated the presentation of each problem.

We presented the entire set of 64 problems (8 practice problems, 48 experimental problems, and 8 control problems) in four sessions lasting approximately 20 minutes each. For each session, half of the problems involved the ED relation, and the other half, the EB relation. The four premises of a set were presented in the predefined order (see Appendix A). However, each subject received a question type and related questions in a different random order to free data from any potential practice and/or sequence effects.

After completing the entire set of problems, the experimenter asked the subject to discuss: (1) how s/he remembered the entities in the premises, (2) how s/he organised and remembered the relative locations of the entities, and (3) how s/he solved the problems. We tape recorded their explanations, considered as a metacognitive activity (Berry, 1983), for further analysis concerning the nature of the mental representation and processes that underlie spatial deductive reasoning (Evans, 1991; Evans, Newstead, & Byrne, 1993; Roberts, 1993).

### 2.3. Experimental Design

The experiment included six within-subject variables [referential determinacy (3) x dimensional condition (2) x orientation (2), direction (2) x premise order (4), question type (2)]. A total of 48 experimental problems, each including four premises, covered the within-subject variables. All problems were displayed according to two between-subject conditions [symbolic structure (2) x symbolic content (2)]. Table 1 presents the between-subject conditions illustrated by a simple example of premise. There were three dependant variables: the premise display times, the responses, and the response times.

Table 1. Between-subject conditions illustrated by a simple example of premise.

Symbolic structure	Symbolic content			
	Nouns		Images	
Sentences	star left of cross		* left of +	
Diagrams	star	cross	*	+

#### 2.4. Subjects

A total of 40 subjects (30 males, 10 females; 18 to 49 years old) of various occupational levels (clerk to scientist), military ranks (private to lieutenant), and educational levels (high school to graduate degree) completed the experiment. Subjects were assigned to one of the four between-subject conditions ( $n = 10$  each) balanced according to the participants' occupational and educational levels. The four conditions were: diagrams-images, diagrams-nouns, sentences-images, sentences-nouns. The results of three subjects, indicating random responses, were eliminated prior to statistical analysis. All subjects were paid according to DCIEM guidelines for stress allowance.

### **3. Statistical Analyses**

We initially carried out three separate analyses of variance (ANOVA) for repeated measures, one on the premise display times (DTs), a second on the percentages of correct responses (CRs), and a third one on the response times (RTs) obtained for the correct responses (Boudreau & Pigeau, 1997). The latter two dependant variables were analysed for the first question of each question type (EB relation and ED relation). The results of these analyses have been summarised in the introduction.

In this study, we will elucidate the temporal consistency of the effects of each independent variable on the percentages of correct responses (CRs) and the response times (RTs) obtained for the successive questions that were asked for the 2D and the 3D conditions. For the 2D and 3D conditions, the CRs and RTs were obtained for the 2D questions related to the EB relation; and for the 3D condition, these dependant measures were obtained for the 3D questions associated to the ED relation. Because the 2D and 3D questions depended on the dimensional condition, for each question type we carried out two separate ANOVAs for repeated measures, one on the RTs obtained for the correct responses, and a second one on the percentages of CRs. The ANOVAs were carried out directly on the RTs which were normally distributed.

For each dependant variable, the design of the ANOVA included two between-subject factors, each with two levels (1) the symbolic structure of the problems, textual versus graphical, and (2) the symbolic content of the problems, nouns versus images.

In addition to the between-subject factors, each ANOVA included the following set of within-subject factors. For the 2D questions of the EB relation, the analysis included five within-subject factors: (1) logical structure (problems of type 1, type 2, and type 3); (2) dimensional condition (2D and 3D); (3) orientation (horizontal and vertical); (4) direction (right, left, bottom, and top), and (5) question order (or the ordinal value Q1, Q2). For the ED relation of the 3D condition, the analysis included four within-subject factors: (1) logical structure (problems of type 1, type 2, and type 3); (2) orientation (horizontal and vertical); (3) direction (right, left, bottom, and top), and (4) question order (or the ordinal value Q1, Q2, Q3).

For each of the above analyses, we performed Geiser-Greenhouse epsilon corrections to adjust the degrees of freedom of each effect. Contrasts between pairs of means were calculated for the levels of the significant effects set at the probability of 0.01.

#### 4. Results and Discussion

The result section will first address the effects of question order on spatial reasoning (section 4.1). We will then assess the temporal consistency of the effects of (a) the symbolic representation of the arguments (section 4.2), (b) their logical structure (section 4.3), and (c) their geometrical content (section 4.4) on the processes of spatial reasoning.

##### 4.1. Effects of Question Order on the Processes of Spatial Reasoning

Figure 1 depicts the mean RTs and the percentages of CRs obtained for each question of each question type. As shown, the first question of both dimensional conditions took significantly more time to answer than the second one [ $F(1, 33) = 59.03, p < 0.01$ ], but the two questions were answered with similar accuracy [ $F(1, 33) = 1.78$ ]. For the 3D condition, question order also had a significant main effect on the RTs [ $F(2, 66) = 36.41, p < 0.01$ ], but not on the CRs [ $F(2, 66) = .89$ ]. Contrasts indicate that the first question took significantly longer to answer than the second one [ $F(1, 33) = 38.36, p < 0.01$ ] and the third one [ $F(1, 33) = 66.91, p < 0.01$ ] respectively, but the latter two did not differ significantly [ $F(1, 33) = 3.94$ ].

The effects of question order on the response times and the correct responses replicate those that we obtained in a previous study on spatial deductive reasoning (Boudreau et al., 1995). In the latter study, question order significantly affected the speed with which subjects answered the 2D questions [RTs:  $F(1, 25) = 108.54, p < 0.01$ ; CRs:  $F(1, 25) = .11$ ], and the 3D ones [RTs:  $F(2, 50) = 65.38, p < 0.01$ ; CRs:  $F(2, 50) = 2.61$ ]. For the 3D questions, contrasts indicate that the first question took reliably longer to answer than the second one [RTs:  $F(1, 25) = 109.27, p < 0.01$ ; CRs:  $F(1, 25) = 5.14, p < 0.02$ ] and the third one [RTs:  $F(1, 25) = 85.40, p < 0.01$ ; CRs:  $F(1, 25) = .77$ ] respectively, but the latter two did not differ significantly [RTs:  $F(1, 25) = 1.47$ ; CRs:  $F(1, 25) = 1.93$ ].

Thus in both studies, the first question of the 2D and 3D conditions was reliably easier to answer than the second one, and the third one (for the 3D condition) suggesting that subjects were consolidating and/or reviewing their mental representation of the spatial relations before deducing a relative location from this

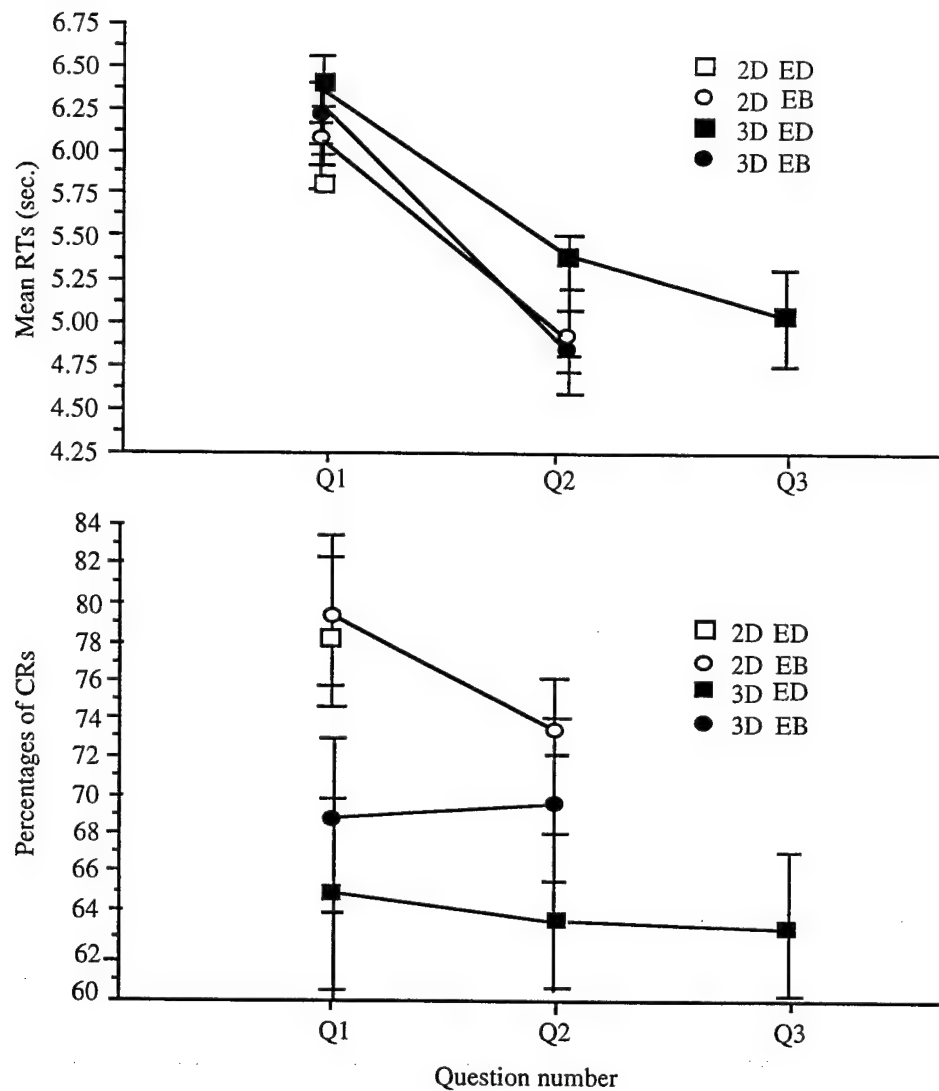


Fig. 1. Mean response times (RTs) and percentages of correct responses (CRs) obtained for the 2D questions (Q1, Q2) requested for the 2D and 3D conditions (EB relation) and the 3D questions (Q1, Q2, Q3) requested for the 3D condition (ED relation)

Table 2. Mean response times (RTs) and percentages of correct responses (% of CRs) obtained for the between-subject conditions, that is, the symbolic structure (diagrams or sentences) and the symbolic content (images or nouns) of the arguments.

Between-subject conditions	EB relation	EB relation	ED relation	ED relation
	(2D questions)	(2D questions)	(3D questions)	(3D questions)
	Mean RTs	% of CRs	Mean RTs	% of CRs
<b>Symbolic Structure</b>				
Diagrams	5.3	72	5.5	60
Sentences	5.8	74	5.8	66
<b>Symbolic Content</b>				
Images	5.7	69	5.8	63
Nouns	5.4	77	5.5	63
<b>Symbolic Structure &amp; Content</b>				
Diagrams-Images	5.2	70	5.2	63
Diagrams-Nouns	5.3	74	5.9	57
Sentences-Images	6.2	67	6.5	63
Sentences-Nouns	5.4	79	5.2	68

mental representation. We will now specify whether the effects of structure and content generalised across the successive questions.

#### 4.2. Temporal Consistency of the Effects of the Arguments' Symbolic Representation

##### 4.2.1. Effects of the Symbolic Structure

For the first question of each question type, deductions from diagrams were reliably easier to make than from sentences [RTs:  $F(1, 33) = 7.08, p < 0.01$ ; CRs:  $F(1, 33) = .33$ ] thus confirming the Mental Models theory. As shown in Table 2, the differences between diagrams and sentences remained consistent although not significant across the successive questions [(2D questions -- RTs:  $F(1, 33) = 2.16$ ; CRs:  $F(1, 33) = 1.06$ ), (3D questions -- RTs:  $F(1, 33) = .50$ ; CRs:  $F(1, 33) = .91$ )]. As illustrated in Figure 2, for the 2D questions, there were no reliable interaction between the symbolic structure of the arguments and the question's ordinal value [RTs:  $F(1, 33) = .05$ ; CRs:  $F(1, 33) = 4.17$ ].

However, for the 3D condition, there was a significant *symbolic structure x question order* interaction [3D questions -- RTs:  $F(2, 66) = 17.00, p < 0.01$ ; CRs:  $F(2, 66) = .04$ ]. As illustrated in Figure 2, the use of diagrams facilitated 3D deductions during the first question [RTs:  $F(1, 33) = 8.43, p < 0.01$ ].

However, there were no reliable differences between sentences and diagrams for the second question [RTs:  $F(1, 33) = 1.69$ ] or the third one (means RTs: 5.1 and 5.0). These results suggest that diagrams facilitate spatial reasoning only during the initial stage of the process of deduction. Once subjects have consolidated their mental models of the spatial relations, their deductions were just as easy to make from sentences than from diagrams.

#### 4.2.2. Effects of the Symbolic Content

The symbolic content of the arguments did not have a significant main effect on spatial deductions either during the first question of the two question types [RTs:  $F(1, 33) = 2.54$ ; CRs:  $F(1, 33) = .47$ ] or throughout the 2D questions [RTs:  $F(1, 33) = 1.06$ ; CRs:  $F(1, 33) = 2.15$ ], and the 3D ones [RTs:  $F(1, 33) = .57$ ; CRs:  $F(1, 33) = .02$ ]. Also, there were no reliable interaction between the symbolic content of the arguments and the question's ordinal value [(2D questions -- RTs:  $F(1, 33) = .23$ ; CRs:  $F(1, 33) = .53$ ), (3D questions -- RTs:  $F(2, 66) = 2.84$ ; CRs:  $F(2, 66) = .51$ )].

For the 3D condition, there was a significant interaction between the symbolic structure and the symbolic content of the arguments [3D questions -- RTs:  $F(1, 33) = 6.32$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = .97$ ], but this interaction did not generalise to the 2D questions [RTs:  $F(1, 33) = 1.20$ ; CRs:  $F(1, 33) = .52$ ]. Separate ANOVAs indicate that deductions were reliably more difficult to make from the sentences-images condition (6.5 sec.) than from the diagrams-images condition (5.2 sec.) depending on the question's ordinal value [RTs:  $F(2, 66) = 8.49$ ,  $p < 0.01$ ]. But this difference occurred only during the first question ( $p < 0.01$ ). Conversely, deductions were more difficult to make from the diagrams-nouns condition (5.9 sec.) than from the sentences-nouns condition (5.2 sec) but the differences were not significant [RTs:  $F(1, 17) = 3.54$ ]. There were no reliable *question order x symbolic content x symbolic structure* interaction [(2D questions -- RTs:  $F(1, 33) = .58$ ; CRs:  $F(1, 33) = .03$ ), (3D questions -- RTs:  $F(2, 66) = 2.54$ ; CRs:  $F(2, 66) = .23$ )]. Thus, as predicted by the Formal Rules theory and the Mental models theory, the use of images within sentences increased the difficulty of the spatial deductions because of the increased mental workload required to solve the incompatibility between the representation of images within sentences. However, neither of the effects of the symbolic structure or the symbolic content persisted across the successive questions.

#### 4.3. Temporal Consistency of the Effects of Logical Structure

As indicated in the introduction, problem type had a significant main effect on the response times during the presentation of the first question ( $p < 0.01$ ). However, the effect of problem type did not remain significant

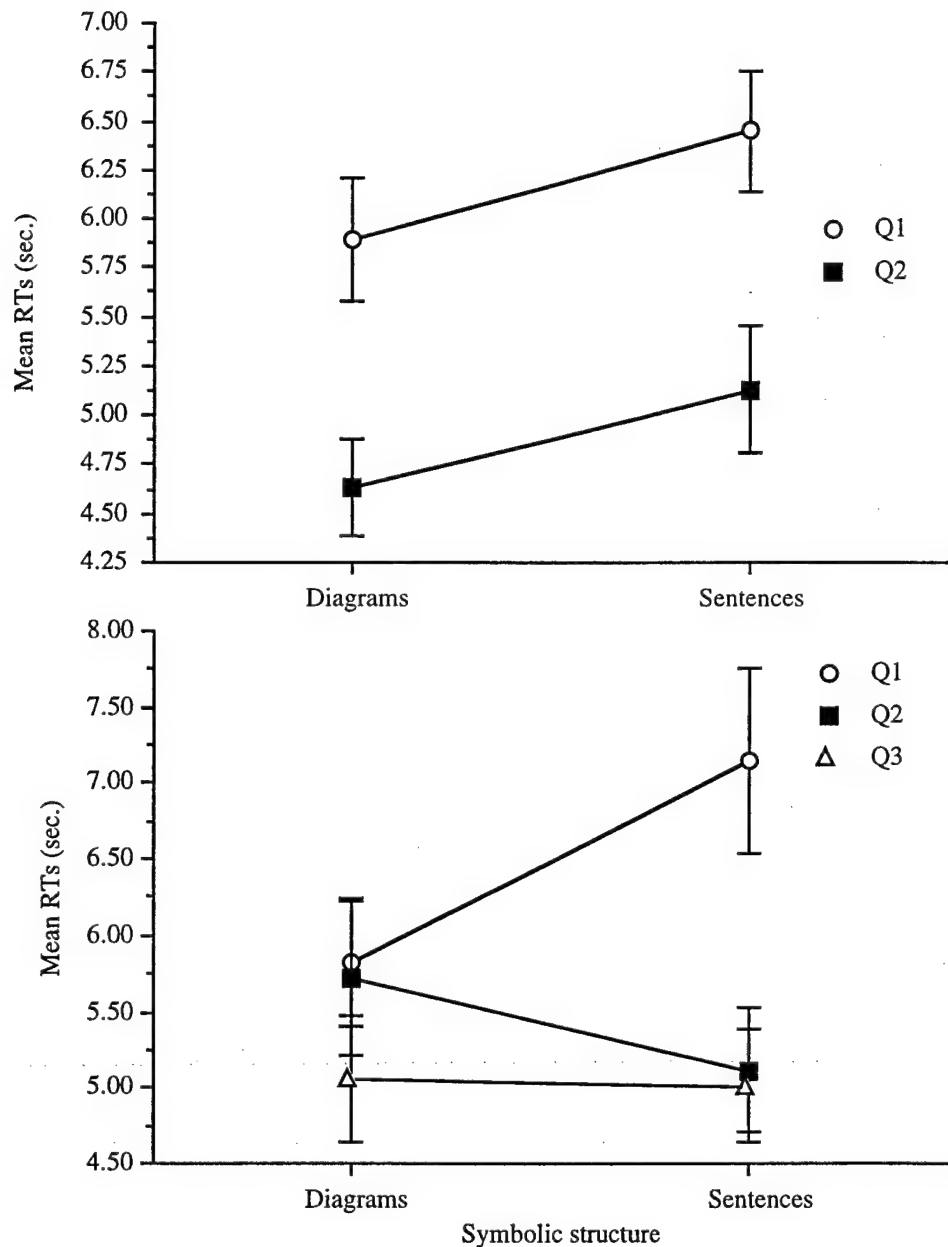


Fig. 2. Mean response times (RTs) obtained for each of the 2D questions (Q1, Q2) of the EB relation and each of the 3D questions (Q1, Q2, Q3) of the ED relation presented either as diagrams or as sentences. The symbols Q1, Q2, and Q3 represent the ordinal value of the questions irrespective of their content.

throughout the 2D questions [RTs:  $F(2, 66) = .04$ ; CRs:  $F(2, 66) = 1.39$ ], or the 3D ones [RTs:  $F(2, 66) = 1.47$ ; CRs:  $F(2, 66) = .66$ ]. For the 3D condition, there was a reliable interaction between problem type and the question's ordinal value [(3D questions -- RTs:  $F(4, 132) = 6.35$ ,  $p < 0.01$ ; CRs:  $F(4, 132) = 1.57$ ), (2D questions -- RTs:  $F(2, 66) = 2.42$ ; CRs:  $F(2, 66) = .07$ )]. Contrasts indicate that for the first

Table 3. Mean response times (RTs) and percentages of correct responses (% CRs) obtained for the within-subject variables, that is, the logical structure (problem type) and geometrical content (dimensional condition, orientation, and direction) of the arguments.

Within-subject variables	EB relation (2D questions)	EB relation (2D questions)	ED relation (3D questions)	ED relation (3D questions)
	Mean RTs	% CRs	Mean RTs	% CRs
<b>Logical Structure</b>				
Problem type 1	5.5	71	5.6	63
Problem type 2	5.5	75	5.8	61
Problem type 3	5.5	72	5.5	65
<b>Geometrical Content</b>				
Dimensional condition				
2D	5.5	77		
3D	5.5	69		
Orientation				
Horizontal	5.4	72	5.7	63
Vertical	5.6	74	5.6	62
Direction				
Right to left	5.0	77	5.6	62
Left to right	5.9	67	5.8	65
Bottom to top	5.5	74	5.3	63
Top to bottom	5.7	73	6.0	61

question of the 3D condition, problem type 2, based on two mental models were reliably more difficult than problem type 1 [ $F(1, 33) = 10.22, p < 0.01$ ] and problem type 3 [ $F(1, 33) = 16.84, p < 0.01$ ] respectively. There were no reliable differences between problems of type 1 and type 3 [ $F(1, 33) = .82$ ].

The effect of problem type did not interact significantly with the symbolic structure of the arguments [(2D questions -- RTs:  $F(2, 66) = .51$ ; CRs:  $F(2, 66) = .69$ ), (3D questions -- RTs:  $F(2, 66) = 3.44$ ; CRs:  $F(2, 66) = 1.34$ ), or their symbolic content [2D questions -- RTs:  $F(2, 66) = .85$ ; CRs:  $F(2, 66) = 1.49$ ] except for the 3D condition [3D questions -- RTs:  $F(2, 66) = 6.08, p < 0.01$ ; CRs:  $F(2, 66) = .64$ ]. Separate analyses were performed to measure the effects of problem type for the images conditions on one hand, and for the nouns conditions on the other hand. For the images conditions, problems type 2 based on two mental models were reliably more difficult than those of type 1 and type 3 based one mental model



Table 4. Mean response times (RTs) and percentages of correct responses (% of CRs) obtained for the 2D questions (EB relation) requested for the problems presented according to the between-subject conditions and the within-subject variables.

Within subject-variables	Between subject-conditions							
	Symbolic structure (Mean RTs)		Symbolic content (Mean RTs)		Symbolic structure (% of CRs)		Symbolic content (% of CRs)	
	Diagrams	Sentences	Images	Nouns	Diagrams	Sentences	Images	Nouns
<b>Logical Structure</b>								
Problem type 1	5.3	5.8	5.8	5.3	68	74	69	73
Problem type 2	5.3	5.7	5.5	5.4	78	71	71	79
Problem type 3	5.2	5.9	5.7	5.4	69	75	67	78
<b>Geometrical Content</b>								
Dimensional condition								
2D	5.4	5.7	5.6	5.4	79	75	73	80
3D	5.2	5.9	5.7	5.3	66	73	65	73
Orientation								
Horizontal	5.1	5.8	5.7	5.2	70	75	69	75
Vertical	5.5	5.8	5.7	5.5	74	73	69	78
Direction								
Right to left	4.6	5.4	5.2	4.8	73	82	75	80
Left to right	5.6	6.2	6.2	5.6	67	67	63	71
Bottom to top	5.6	5.4	5.4	5.7	76	73	70	78
Top to bottom	5.3	6.1	6.1	5.3	73	73	68	78

[type 2 vs. type 3:  $F(1, 32) = 9.86, p < 0.01$ ; type 2 vs. type 1:  $F(1, 32) = 1.02$ ]. But these differences occurred only during the first question. When nouns were used, there were no significant differences between the three problem types [ $F(2, 34) = 1.81$ ].

The fact that multiple mental models increased the difficulty of spatial reasoning only during the first question suggests two possible interpretations. The first is that subjects were capable of holding and inspecting two mental models in working memory only during the time required to answer the first question of each question type. The second is that, for the subsequent questions, subjects may have overlooked alternative mental models having discovered that a conclusion was valid in both mental models.

Table 5. Mean response times (RTs) and percentages of correct responses (% of CRs) obtained for the 3D questions (ED relation) requested for the problems presented according to the between-subject conditions and the within-subject variables.

Within subject-variables	Between-subject variables							
	Symbolic structure		Symbolic content		Symbolic structure		Symbolic content	
	(Mean RTs)		(Mean RTs)		(% of CRs)		(% of CRs)	
	Diagrams	Sentences	Images	Nouns	Diagrams	Sentences	Images	Nouns
<b>Logical</b>								
<b>Structure</b>								
Problem type 1	5.2	5.9	5.9	5.3	57	69	61	65
Problem type 2	5.7	5.9	6.2	5.5	61	60	63	58
Problem type 3	5.7	5.3	5.2	5.8	62	68	64	65
<b>Geometrical</b>								
<b>Content</b>								
<b>Orientation</b>								
Horizontal	5.4	5.9	5.9	5.4	59	68	63	64
Vertical	5.7	5.6	5.6	5.7	61	63	63	61
<b>Direction</b>								
Right to left	5.4	5.7	5.7	5.4	56	68	61	62
Left to right	5.4	6.1	6.2	5.3	63	68	65	65
Bottom to top	5.5	5.1	5.1	5.4	61	66	70	57
Top to bottom	5.8	6.1	6.1	5.9	61	61	56	66

#### 4.4. Temporal Consistency of the Effects of Geometrical Content

##### 4.4.1. Effects of Dimensional Condition

For the first question of both question types, spatial deductions were reliably easier to make from the 2D condition than the 3D one [CRs:  $F(1, 33) = 15.48, p < 0.01$ ; RTs:  $F(1, 33) = 9.46, p < 0.01$ ]. The differences between the 2D and 3D conditions also remained consistent and significant throughout the successive questions [2D questions: CRs:  $F(1, 33) = 11.11, p < 0.01$ ; RTs:  $F(1, 33) = .16$ ].

As shown in Table 4, the effect of dimensional condition interacted significantly with the symbolic structure of the arguments [CRs:  $F(1, 33) = 7.61, p < 0.01$ ; RTs:  $F(1, 33) = 1.48$ ]. Using sentences, the 2D (75%) and 3D conditions (73%) were solved with similar accuracy [CRs:  $F(1, 16) = .26$ ]; but using diagrams, the 2D condition (79% correct) was significantly easier than the 3D one (66% correct) [CRs:  $F(1, 17) = 13.99, p < 0.01$ ].

The differences between the 2D and 3D conditions generalised across the symbolic content of the arguments [RTs:  $F(1, 33) = .31$ ; CRs:  $F(1, 33) = .05$ ]. However, the effect of the dimensional conditions on the RTs interacted reliably with the arguments' symbolic structure and symbolic content [RTs:  $F(1, 33) = 4.63$ ,  $p < 0.03$ ; CRs:  $F(1, 33) = .36$ ]. The interaction is related to the fact that for the sentences-images condition, deductions took significantly more time to make from the 3D layouts than from the 2D ones (see Figure 3).

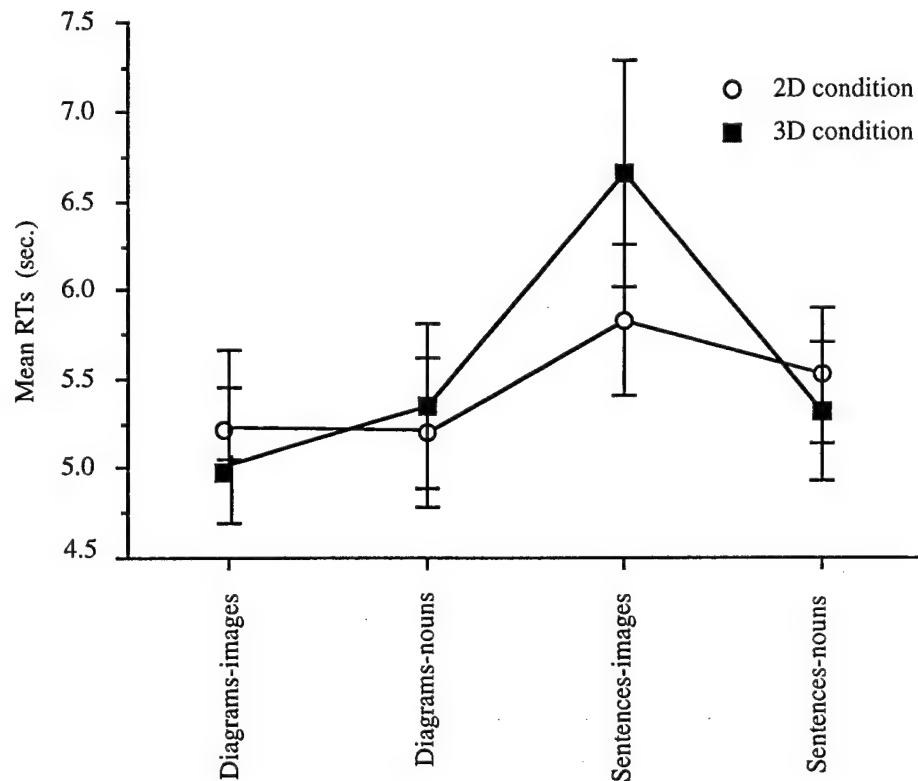


Fig. 3. Mean response times (RTs) obtained for the EB relation of each dimensional condition symbolised either as diagrams or as sentences using either images or nouns

The effect of the dimensional conditions remained consistent across problem type [RTs:  $F(2, 66) = .61$ ; CRs:  $F(2, 66) = 1.60$ ], orientation [RTs:  $F(1, 33) = 1.67$ ; CRs:  $F(1, 33) = .19$ ], and direction [RTs:  $F(1, 33) = 1.77$ ; CRs:  $F(1, 33) = .001$ ]. None of the other three-way interactions between the dimensional conditions and any two variables were reliable ( $p > 0.01$ ).

#### 4.4.2. Effects of Orientation

As indicated in the introduction, the horizontal layouts were significantly easier to represent than the vertical layouts ( $p < 0.01$ ). However, orientation did not have a significant main effect on spatial deductions either during the first question of each question type [RTs:  $F(1, 33) = 2.43$ ; CRs:  $F(1, 33) = .34$ ], the 2D

questions [RTs:  $F(1, 33) = .64$ ; CRs:  $F(1, 33) = .21$ ], or the 3D ones [RTs:  $F(1, 33) = .001$ ; CRs:  $F(1, 33) = .20$ ]. For the 3D condition, orientation interacted significantly with the question's ordinal value [(3D questions -- RTs:  $F(2, 66) = 6.14$ ,  $p < 0.01$ ; CRs:  $F(2, 66) = .24$ ), (2D questions -- RTs:  $F(1, 33) = .40$ ; CRs:  $F(1, 33) = .39$ )]. Contrasts indicate that the vertical layouts were reliably more difficult than the horizontal ones for the first question [RTs:  $F(1, 33) = 8.13$ ,  $p < 0.01$ ], but not for the second one [RTs:  $F(1, 33) = 3.08$ ] or the third one [RTs:  $F(1, 33) = 1.07$ ].

There was a significant *orientation x direction* interaction for the 2D questions [RTs:  $F(1, 33) = 6.58$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = 2.97$ ] but not for the 3D ones [RTs:  $F(1, 33) = 2.48$ ; CRs:  $F(1, 33) = 1.03$ ]. As shown in Table 4, layouts described from bottom to top along the vertical axis were significantly more difficult than those described from right to left along the horizontal axis [RTs:  $F(1, 33) = 5.95$ ,  $p < 0.01$ ]. There were no differences between the two orientations when layouts were described from top to bottom or from left to right [RTs:  $F(1, 33) = 1.41$ ].

The differences in difficulty between the two orientations remained consistent across the symbolic structure [(2D questions -- RTs:  $F(1, 33) = 2.68$ ; CRs:  $F(1, 33) = 1.28$ ), (3D questions -- RTs:  $F(1, 33) = 3.35$ ; CRs:  $F(1, 33) = .82$ )], and the symbolic content of the arguments [(2D questions -- RTs:  $F(1, 33) = 1.22$ ; CRs:  $F(1, 33) = .37$ ), (3D questions -- RTs:  $F(1, 33) = 3.63$ ; CRs:  $F(1, 33) = .02$ )]. However, for the 3D condition, the interaction between the latter three factors was reliable [(3D questions -- RTs:  $F(1, 33) = 11.13$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = .05$ ), (2D questions -- RTs:  $F(1, 33) = 4.18$ ; CRs:  $F(1, 33) = .03$ )]. Separate analyses were performed for the sentences conditions on one hand and for the diagrams conditions on the other hand. As illustrated in Figure 4, the 3D horizontal layouts and the 3D vertical layouts were both more difficult to construct in the sentences-images condition than in the sentences-nouns condition [RTs:  $F(1, 16) = 4.23$ ,  $p < 0.05$ ]. The 3D vertical layouts were more difficult to construct than the 3D horizontal layouts in the diagrams-nouns condition than in the diagrams-images condition [RTs:  $F(1, 17) = 13.20$ ,  $p < 0.01$ ], but the horizontal layouts were just as easy to construct in either condition.

The above results suggest that for the 2D and 3D conditions, there were no main effect of orientation on the difficulty of spatial deduction. However for both conditions, the vertical layouts were more difficult to construct than the horizontal ones when the former were specified from bottom to top along the vertical axis. For the 3D condition, the vertical layouts were reliably more difficult to construct in the diagrams-nouns conditions than in the diagrams-images condition. But in the sentences-images condition the horizontal layouts were as difficult to construct than the vertical ones.

#### 4.4.3. Effects of Direction

The effect of direction was temporally consistent as it remained significant throughout the successive 2D questions [RTs:  $F(1, 33) = 11.35$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = 6.08$ ,  $p < 0.01$ ], and the 3D ones [RTs:  $F(1, 33) = 7.62$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = .02$ ]. The effects of direction also remained consistent across the question's ordinal value [(2D questions -- RTs:  $F(1, 33) = .84$ ; CRs:  $F(1, 33) = .25$ ), (3D questions --

RTs:  $F(2, 66) = 2.58$ , CRs:  $F(2, 66) = .84$ ], the dimensional conditions [2D questions -- RTs:  $F(1, 33) = 1.77$ ; CRs:  $F(1, 33) = .001$ ] and, overall, both orientations [(2D questions -- RTs:  $F(1, 33) = 6.58$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = 2.97$ ), (3D questions -- RTs:  $F(1, 33) = 2.48$ ; CRs:  $F(1, 33) = 1.03$ ]]. As shown in Table 4, spatial deductions were significantly easier to make from right to left along the horizontal axis [(2D questions -- RTs:  $F(1, 33) = 20.46$ ,  $p < 0.01$ ; CRs:  $F(1, 33) = 6.60$ ,  $p < 0.01$ ), (3D questions -- RTs:  $F(1, 33) = 1.01$ )], and from bottom to top along the vertical axis [(2D questions -- RTs:  $F(1, 33) = .80$ ; CRs:  $F(1, 33) = .10$ ), (3D questions -- RTs:  $F(1, 33) = 10.42$ ,  $p < 0.01$ )] rather than from the opposite set of directions.

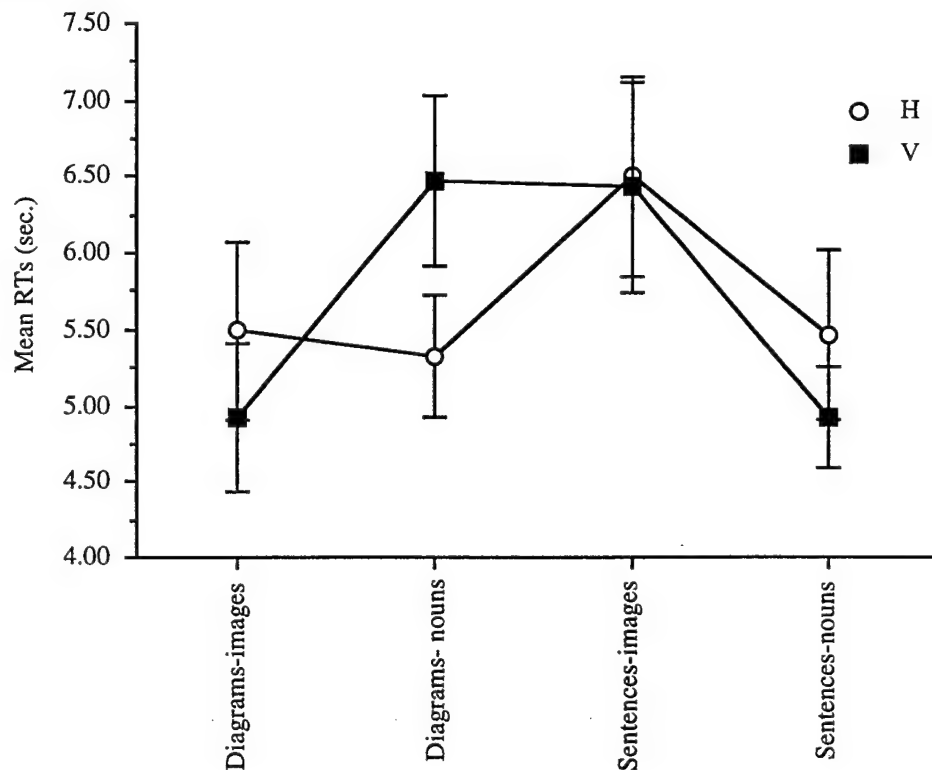


Fig. 4. Mean response times obtained for the 3D horizontal layouts (H) and the 3D vertical layouts (V) displayed either as sentences or as diagrams using either nouns or images.

These results replicate those that we obtained (Boudreau et al. 1995) in a previous study on spatial deductive reasoning. In the latter study, layouts were reliably easier to construct when described from right to left along the horizontal axis and from bottom to top along the vertical axis rather than from the opposite set of directions [(2D questions -- RTs:  $F(1, 25) = 17.63$ ,  $p < 0.01$ ; CRs:  $F(1, 25) = 12.39$ ,  $p < 0.01$ ), (3D questions -- RTs:  $F(2, 50) = 24.41$ ,  $p < 0.01$ ; CRs:  $F(2, 50) = 2.48$ )].

The effects of direction were consistent across problem type [(2D questions -- RTs:  $F(2, 66) = .79$ ; CRs:  $F(2, 66) = .56$ ), (3D questions -- RTs:  $F(2, 66) = .20$ ; CRs:  $F(2, 66) = 1.08$ )], the symbolic structure [(2D questions -- RTs:  $F(1, 33) = 2.22$ ; CRs:  $F(1, 33) = .23$ ), (3D questions -- RTs:  $F(1, 33) = 3.06$ ; CRs:

$F(1, 33) = 1.66$ ], and the symbolic content of the arguments [(2D questions -- RTs:  $F(1, 33) = 4.37$ ; CRs:  $F(1, 33) = .21$ ), (3D questions -- RTs:  $F(1, 33) = 2.75$ ; CRs:  $F(1, 33) = 4.92$ )].

### 5. Conclusions

The general aim of this study was to elucidate the temporal consistency of the effects of the structure and content of an argument on the processes of spatial deductive reasoning. The effect of the symbolic structure of the arguments indicates that spatial deductions were reliably easier to make from diagrams than from sentences, thus supporting the Mental Models theory. The differences between diagrams and sentences remained consistent, although not significant, across the successive questions of the 2D and 3D conditions. The interaction between the symbolic structure and the question's ordinal value suggests that diagrams facilitated spatial reasoning only during the initial stage of the process of deduction. Once subjects had consolidated their mental models of the spatial relations, their deductions were just as easy to make from sentences than from diagrams.

The symbolic content of the questions did not have a reliable effect on spatial deductions either during the first question of each question type or throughout the successive ones ( $p > 0.01$ ). Also, there were no reliable interaction between the symbolic content and the question's ordinal value. Thus, as predicted by the Formal Rules theory and the Mental models theory, the effect of the symbolic structure of the arguments prevailed over the lexical tokens, such as nouns or images, used to symbolise the arguments' entities. However, the fact that spatial deductions were reliably easier to make from diagrams than from sentences corroborates the Mental Models theory.

As indicated in our basic study (Boudreau & Pigeau, 1997), problem type had a significant main effect on the response times during the presentation of the first question ( $p < 0.01$ ). However, the effect of problem type did not remain significant throughout the successive questions ( $p > 0.01$ ). The results suggest two possible interpretations. The first is that subjects were capable of holding and inspecting two mental models in working memory only during the time required to answer the first question of each question type. The second is that after having discovered that a conclusion was valid for both mental models, subjects may have overlooked the alternative mental model.

Spatial deductions were reliably easier to make from the 2D condition than the 3D one for the first question of both question types ( $p < 0.01$ ). The differences between the 2D and 3D conditions also remained significant throughout the successive questions ( $p < 0.01$ ) and were thus temporally consistent.

Mental representation were significantly easier to construct from the horizontal layouts than from the vertical layouts ( $p < 0.01$ ). However, orientation did not have a significant main effect on spatial deductions either during the first question or throughout the successive ones ( $p > 0.01$ ). For both dimensional conditions, the interaction between orientation and direction suggests that the vertical layouts were more difficult to construct than the horizontal ones when the former were specified from bottom to top along the vertical axis, and the latter from right to left.

The effects of direction on spatial deductions were temporally consistent as they remained significant throughout the successive 2D and 3D questions ( $p < 0.01$ ). As predicted by the Content-Specific rules theory (DeSoto et al., 1965), subjects had directional preferences in constructing mental models from one set of directions rather than from opposite ones. Spatial deductions were significantly easier to make from right to left along the horizontal axis ( $p < 0.01$ ) and from bottom to top along the vertical axis ( $p < 0.01$ ) rather than from the opposite set of directions. The effects of the geometrical relations (dimensions, orientation, and directions) elucidated the structural properties of mental models. The results suggest that mental models reproduce the euclidan and projective relations among the entities in a way that is similar to which mental images reproduce metric relations among entities. The consistent effects of the geometrical relations indicate that mental models reproduce these relations according to spatial reference frames (Logan, 1995; Herskovits, 1986).

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## Appendix A

# Appendix A

Appendix A contains the 48 experimental problems. It describes the variables used to create the premise sets, that is, logical structure [problem type (3 levels)], and geometrical content [dimensional condition (2 levels), orientation (2 levels), direction (2 levels)]. The appendix also describes the variables nested within the question types [number of dimensions (3 levels)]. The difficulty of the problems yields two sets of predictions, one based on the number of mental models and the other on the number of inferential steps in the formal derivation of a relative location.

Logical structure	Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Problem type 1	Horizontal layouts and directions		Dimensional Condition 2D Vertical layouts and directions		EB relation 2D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
	H1	H2	V1	V2	Examples	Examples		2 Steps	4 Steps
A r B	A right of B	A left of B	A below B	A above B	E left of B?	E left of D?	1	1: $r(CE) \leftrightarrow -r(EC)$	1: $r(CE) \leftrightarrow -r(EC)$
C r B	C left of B	C right of B	C above B	C below B	E below B?			2: $r(EC) \ \& \ r(CB) \Rightarrow r(EB)$	2: $r(EC) \ \& \ r(CB) \Rightarrow r(EB)$
D r B	D below B	D above B	D left of B	D right of B				$r(EB)$	3: $r(DB) \leftrightarrow -r(BD)$
CrE	C above E	C below E	C right of E	C left of E				4: $r(EB) \ \& \ r(BD) \Rightarrow r(ED)$	
Mental layouts	CB A ED	DE ABC	EC DB A	A BD CE					

(Appendix A continues)

Logical structure	Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Problem type 2	Dimensional Condition 2D Horizontal layouts and directions Vertical layouts and directions				EB relation 2D	ED relation 1D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
BrA CrB DrB CrE	H1	H2	V1	V2	Examples	Examples	2	2 Steps 1: $r(CE) \leftrightarrow -r(EC)$ 2: $r(EC) \& r(CB) \Rightarrow r(EB)$	4 Steps 1: $r(CE) \leftrightarrow -r(EC)$ 2: $r(EC) \& r(CB) \Rightarrow r(EB)$ 3: $r(DB) \leftrightarrow -r(BD)$ 4: $r(EB) \& r(BD) \Rightarrow r(ED)$
	B right of A	B left of A	B below A	B above A	E left of B?	E left of D?			
	C left of B	C right of B	C above B	C below B	E below B?				
	D below B	D above B	D left of B	D right of B					
Mental layouts	C above E	C below E	C right of E	C left of E					
	CAB E D ACB ED	D E BAC DE BCA	EC A A EC DB DB	BD BD A CE CE A					

(Appendix A continues)

Logical structure	Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Problem type 3	Horizontal layouts and directions		Dimensional condition 2D		EB relation	ED relation		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
					2D	1D			
	H1	H2	V1	V2	Examples	Examples		1	2 Steps
Ar B	A right of B	A left of B	A below B	A above B	E left of B?	E left of D?	1	1: $r(CE) \leftrightarrow -r(EC)$	1: $r(CE) \leftrightarrow -r(EC)$
Cr B	C left of B	C right of B	C above B	C below B	E below B?			2: $r(EC) \ \& \ r(CB) \Rightarrow r(EB)$	2: $r(EC) \ \& \ r(CB) \Rightarrow r(EB)$
Dr A	D below A	D above A	D left of A	D right of A				$r(EB)$	3: $r(AB) \leftrightarrow -r(BA)$
Cr E	C above E	C below E	C right of E	C left of E					4: $r(EB) \ \& \ r(BA) \Rightarrow r(EA)$
Mental layouts	CBA E D		D E ABC		E C B DA			5: $r(DA) \leftrightarrow -r(AD)$	
								6: $r(EA) \ \& \ r(AD) \Rightarrow r(ED)$	

(Appendix A continues)

Logical structure	Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Problem type 1	Horizontal layouts and directions		Dimensional Condition 3D		Vertical layouts and directions			Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
	H1	H2	V1	V2					
ArB	A right of B	A left of B	A below B	A above B	Examples	E left of D?	1	2 Steps 1: $r(CE) \leftrightarrow -r(EC)$ 2: $r(EC) \ \& \ r(CB) \Rightarrow r(EB)$	4 Steps 1: $r(CE) \leftrightarrow -r(EC)$ 2: $r(EC) \ \& \ r(CB) \Rightarrow r(EB)$ 3: $r(DB) \leftrightarrow -r(BD)$ 4: $r(EB) \ \& \ r(BD) \Rightarrow r(ED)$
CrB	C left of B	C right of B	C above B	C below B	E left of B?	E below D?			
DrB	D in front of B	D behind B	D in front of B	D behind B	E below B?	E behind D?			
CrE	C above E	C below E	C right of E	C left of E					
Mental layouts	<div>CD A E</div>	<div>E A B C</div>	<div>E C D A</div>	<div>A B C E</div>					

(Appendix A continues)

Logical Structure	Geometrical content	Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
		EB relation 2D	ED relation 3D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
Problem type 2	Dimensional Condition 3D Vertical layouts and directions					
BrA CrB DrB CrE	Horizontal layouts and directions					
	H1					
	H2					
	V1					
Mental layouts	H1	B right of A C left of B D in front of B C above E	B left of A C right of B D behind B C below E			
	H2					
	V1	B below A C above B D in front of B C right of E	B above A C below B D behind B C left of E			
	V2					
		Examples E left of B? E below B?	Examples E left of D? E below D? E behind D?	2	2 Steps 1: $r(CE) \leftarrow -r(EC)$ 2: $r(EC) \& r(CB) \Rightarrow r(EB)$	4 Steps 1: $r(CE) \leftarrow -r(EC)$ 2: $r(EC) \& r(CB) \Rightarrow r(EB)$ 3: $r(DB) \leftarrow -r(BD)$ 4: $r(EB) \& r(BD) \Rightarrow r(ED)$

(Appendix A continues)

Logical structure	Geometrical content				Question type & number of dimensions requested in question type		Number of mental models	Inferential steps in the formal derivation of the EB and ED relations	
Problem type 3	Dimensional Condition 3D Horizontal layouts and directions Vertical layouts and directions				EB relation 2D	ED relation 3D		Inferential steps in the formal derivation of the EB relation	Inferential steps in the formal derivation of the ED relation
A r B C r B D r A C r E	H1	H2	V1	V2	Examples	Examples	1	2 Steps 1: r(CE) <-> -r(EC) 2: r(EC) & r(CB) => r(EB) r(EB)	6 Steps 1: r(CE) <-> -r(EC) 2: r(EC) & r(CB) => r(EB) 3: r(AB) <-> -r(BA) 4: r(EB) & r(BA) => r(EA) 5: r(DA) <-> -r(AD) 6: r(EA) & r(AD) => r(ED)
	A right of B	A left of B	A below B	A above B	E left of B?	E left of D?			
	C left of B	C right of B	C above B	C below B	E below B?	E below D?			
	D in front of A	D behind A	D in front of A	D behind A		E behind D?			
C r E	C above E	C below E	C right of E	C left of E					
Mental layouts	C B D E	E A B C	E C B D	A B C E					

Note 1. The letter "r" represents a relation described in a premise or deduced from an inferential step. The sign -r represents the inverse of a relation specified between an object pair, such as -right(BA) is equivalent to left(AB). The symbol <-> and => denote a relation of equivalence and implication respectively.



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13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

The general aim of this study is to elucidate the temporal consistency of the effects of the structure and content of an argument on spatial deductive reasoning. We will investigate the effects of these factors mainly by comparing the opposing predictions made by the Formal Rules theory and the Mental Models theory of spatial deductive reasoning.

Thirty-seven subjects solved 48 spatial reasoning problems which varied by their *logical structure*, that is, by the order of the entities in the premise sets, and their *geometrical content*, that is, by the number of dimensions (2D, 3D), orientations (horizontal, vertical), and directions (right/left, bottom/top) specified in the premise sets. For half of the problems, subjects were to deduce the relative location of an object pair (EB) in two dimensions. For the other half, subjects were to deduce the relative location of an other object pair (ED) in either one dimension, for the 2D condition, or in three dimensions for the 3D condition. Questions pertaining to each dimension were asked consecutively thus requiring deductions over time but within the bounds of working memory.

We varied the symbolic representation of the problems according to their *symbolic structure* using either sentences or diagrams to represent the relations among entities, and their *symbolic content* using either nouns or images to represent the entities. We addressed temporal consistency by measuring the effects of the above variables on the responses and the response times obtained for the consecutive questions.

Spatial deductions were reliably easier to make from diagrams than from sentences, thus confirming the Mental Models theory. However, diagrams facilitated spatial reasoning only during the initial stage of the process of deduction. The effects of the dimensional conditions and the directions on spatial deductions remained significant and consistent throughout the successive questions. The results suggest that subjects constructed mental models which reproduced the euclidan and projective relations among the entities according to spatial reference frames. Altogether, the effects of the structure (symbolic and logical) and content (symbolic and geometrical) of an argument confirmed the Mental Models theory's predictions while refuting those of the Formal Rules theory.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible, keywords should be selected from a published thesaurus, e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

spatial deductive reasoning, symbolic representation, Mental Models Theory, Formal Rules Theory